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Price Level Convergence, Purchasing Power Parity and Multiple Structural Breaks in Panel Data Analysis: An Application to U.S. Cities

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Abstract

This article provides a methodological and empirical approach for assessing price level convergence and its relation to purchasing power parity (PPP) using annual price data for seventeen U.S. cities during the period 1918 to 2005. We suggest a new panel data procedure that can handle a wide range of PPP concepts in the presence of multiple structural breaks using all possible pairs of real exchange rates. Testing for PPP requires the definition of parametric restrictions (parity restrictions) across regimes. In general, we find more evidence for stationarity when the parity restriction is not imposed, while imposing parity restriction leads toward the rejection of the panel stationarity. Our results can be embedded in the view of the Balassa-Samuelson approach, but where the slope of the time trend is allowed to change in the long-run.

KEYWORDS: price level convergence, pairwise analysis, panel data, multiple structural breaks, cross-section dependence

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1 Introduction

This paper takes a fresh look at price level convergence and Purchasing Power Parity (PPP) hypothesis using annual price data from 1918 to 2005 for seventeen major US cities. Understanding the dynamics of price differentials within a single currency area, such as the US, is relevant not only for its own sake, but also because it offers insights for policymakers in the Euro area, which is a comparatively a young currency union than the US.[1,2]

For many years (reduced) price dispersion has been used as a proxy for (increased) economic integration – and there are potentials gains to be had from the integration of markets and the compression of price divergence. For instance, if local price for an item in Boston is lower than rest of the US, market integration and price convergence will benefit local producers and workers more than they will harm local consumers. The opposite happens when Boston's local price is higher than rest of the US.[3]

Recognizing this potential gain, over the last few years there has been a flurry of papers analyzing the dynamics of price level convergence within a single monetary union. Examining a panel of 51 prices from 48 US cities, Parsley and Wei (1996) find that price convergence occurs much faster among US cities than across national borders. Using long-term series of consumer price indices for 19 major cities over the period from 1918 to 1995, Cecchetti, Mark and Sonora (2002) find that price index divergence across US cities are temporary but surprisingly persistent, with a half-life of nearly nine years. Chen and Devereux (2003) extend the analysis by examining absolute price level dispersion[4] for 19 US cities from 1918 to 2000. They find that price level dispersion is much lower for US cities than OECD countries.[5]

[1] There are some economic interests in studying price divergence within a monetary union. Sustained price divergence not only dampens the law of one price or PPP, it may interfere with the price stability goal of the monetary authority. In addition, significant price differentials can give rise to real interest rate differences and may widen the gap between market exchange rates and PPP exchange rates across markets within a region.

[2] Knowing how large the EU regional price indices deviate from the Euro-wide-area average price index is crucial for the European Central Bank (ECB), who's objective is to fix the year-on-year change in the Harmonized Index of Consumer Prices (HICP) at around 2%.

[3] See Warren, Hufbauer and Wada (2002) for an excellent account on the potential benefits of price level convergence.

[4] Price indices that measure the cost of a given consumption basket at each point in time.

[5] Several studies have examined the dynamics of price level convergence across European cities using shorter span and disaggregated price data. Rogers (2007) finds strong evidence of price level convergence in Europe, especially for traded goods, over the period 1990 to 2001. Crucini et al. (2005) use actual prices for a large sample of items in European cities,

We extend this literature by examining price level convergence for selected US cities in the presence of structural breaks. Accounting for parameter shifts is crucial especially when using long spans of data that are more likely to be affected by structural breaks. The structural breaks can appear either because the data have been sampled across several different monetary arrangements or by the presence of shocks such as oil price shocks. Moreover, aggregate price data are more susceptible to structural instability, because unlike individual price level aggregate price levels may not adjust so quickly due to differences in productivity among traded and non-traded goods sectors (i.e., the Balassa-Samuelson effect). It is therefore important to consider structural breaks in the analysis when using long span of data.[6]

Besides the issue of structural breaks, the notion of convergence used in testing price level convergence is equally important. Most empirical studies have tested the PPP hypothesis using either the average of the price levels or the price level of the leading individual in the sample – for instance, when dealing with international data sets the US price level is usually taken as the reference. The main drawback of this approach is that the results can be dependent on the choice of the benchmark and, as a result, can lead to misleading conclusions. To overcome this limitation, we use pairwise tests for PPP following Pesaran (2007a) and Pesaran et al. (2007), which is not sensitive to the choice of the benchmark. If we analyze N time series of prices, the pairwise tests focus on all possible $N(N-1)/2$ real exchange rate pairs between the time series in the panel and can consistently estimate the proportion of pairs that do not satisfy the PPP. The pairwise approach allows us to obtain more robust results than the ones based on a particular benchmark time series.

Combining these issues, in this paper we propose an econometric framework that can be used to analyze the stochastic nature of price variation and how it relates to the fulfillment of the PPP hypothesis. In particular, we propose new univariate and panel data statistics that account for the presence of multiple structural breaks while simultaneously entertaining the restrictions that impose the PPP hypothesis on the parameters of the model. The consideration of these parametric restrictions (parity restrictions) are required if the analysis wants to test the presence of PPP among the time series. The definition of these parity restrictions requires to impose parametric restrictions among the different regimes that define the structural breaks, which is done

looking at 1975, 1980, 1985, and 1990. They find that PPP holds to a good approximation.

[6] The consequence of neglecting structural breaks on statistical inference is well known. Perron (1989) showed that erroneous omission of structural breaks in the data can bias the inference towards the I(1) non-stationarity conclusion.

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using the approach in Perron and Qu (2006). More specifically, the proposed methodology can handle a wide range of PPP concepts depending on whether real exchange rate evolves around a constant or around a time trend – further details are given below.

The rest of the paper is structured as follows. Section 2 presents a brief account of the various PPP concepts used in this paper. An overview of the econometric methodologies is outlined in Section 3. Section 4 analyzes the finite sample performance of the test through a Monte Carlo experiment. Section 5 presents the data and main empirical results. Concluding remarks appear in Section 6. All proofs are relegated to the appendix.

2 Price convergence, PPP and breaks

This section summarizes the different definitions and concepts that can arise when dealing with price convergence in the presence of structural breaks and how this relates to the PPP hypothesis. This is quite important provided that most of the papers that focus on the PPP hypothesis do not account for the presence of structural breaks and, more interestingly, those that consider structural breaks do not test for the real definition of PPP. Therefore, we believe that the discussion presented in this section can help to disentangle whether price convergence occurs and whether it implies the fulfilment of the PPP hypothesis when multiple structural breaks are considered.

There is a flurry of papers in the economics literature that have investigated whether price convergence has taken part among individuals such as countries, regions or cities focusing on the time series $y_{i,j,t} = (\ln p_{i,t} - \ln p_{j,t})$, that is, the difference between the logarithm of the price of one individual $(p_{i,t})$ and the logarithm of the price of the benchmark individual $(p_{j,t})$, $i, j = 1, \ldots, N$, and $t = 1, \ldots, T$. Since our framework restricts to cities inside a country, $y_{i,j,t}$ can be seen as the real exchange rate provided that US cities share the same currency. The investigation of price convergence is mainly addressed through the assessment of the stochastic properties of the real exchange rates using unit root and stationarity statistics. When real exchange rates are characterized as I(0) stationary processes, it is said that there is evidence in favor of the PPP hypothesis. However, the literature has defined two different concepts of PPP depending on whether real exchange rate evolves around a constant mean or around a linear time trend. Thus, when the deterministic component that is used in the computation of the unit root and stationarity tests is given by a constant term we are dealing with Cassel's (1918) definition of the PPP. Balassa (1964) and Samuelson (1964) devise a second concept of PPP when noticing that divergent international productivity lead to permanent deviations from the Cassel's PPP concept. This feature is captured through the specification of a long-run trend around which the real exchange rates would show stationary fluctuations, which defines the so-called "Trend PPP" (TPPP). Therefore, in this case unit root and stationarity test statistics have to use a linear time trend as the deterministic component when testing for TPPP.

Note that evidence in favor of either PPP or TPPP requires real exchange rate to be I(0). However, misspecifications in the deterministic component of the models in which the unit root and stationarity statistics are based can lead to misleading conclusions. In this regard, Perron (1989) and Lee, Huang and Shin (1997) showed that lack of accounting for the presence of structural breaks can bias the inference towards the I(1) non-stationarity conclusion – see Perron (2006) for a detailed overview. This feature has provoked, firstly, the introduction of structural breaks in the studies that analyze the order of integration of the real exchange rates and, secondly, the definition of new concepts of PPP that are compatible with the presence of structural breaks. To this end, Dornbusch and Vogelsang (1991) consider the presence of one structural break affecting the level of the real exchange rate and coin the term "Qualified PPP" (QPPP) to cover those situations in which real exchange rate is stationary around a changing deterministic component. One relevant feature is that Dornbusch and Vogelsang (1991) interpret this situation as evidence in favor of the Balassa-Samuelson model so that the inclusion of structural breaks can be nested in one of the accepted concepts of PPP in the literature. Other analyses have considered the presence of level shifts when testing the order of integration of real exchange rates with the deterministic specification given either by a constant – see, for instance, Perron and Vogelsang (1992), Hegwood and Papell (1998), and Gadea, Montañés and Reyes (2004) – or by a linear time trend – see Im, Lee and Tieslau (2005), and Papell and Prodan (2006). Following the convention established in Papell and Prodan (2006), we denote by QPPP those situations in which the real exchange rate evolves around a deterministic component given by a constant term with level shifts. Similarly, we denote by "Trend Qualified PPP" (TQPPP) the situation in which the real exchange rate evolves around a deterministic component given by a linear time trend with level shifts.

In fact, the TQPPP definition can accommodate other specifications than the ones defined above. For example, it is possible that events such as the oil embargo or shocks affecting the technological process may change the productivity of individuals in different ways. Thus, divergences in productivity can be reduced or increased after the shocks, which may imply a change in the

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slope of the long-run trend around which the real exchange rates would show stationary fluctuations. This feature can be accounted for including structural breaks that affect both the level and the slope of the time trend. Economically, the presence of structural breaks can be argued from the fact that productivity shocks may have affected traded and non-traded goods sectors differently. In this regard and focusing on the US economy, Bernard and Jones (1996, Table 1) show that labor productivity gains in the traded-good sectors (e.g., mining and manufacturing) have been greater than the productivity gains in the non-traded-good sectors (e.g., construction and service) between 1963 and 1989. They further find that the variation in productivity levels across sectors is consistent with a large amount of variation in productivity across states. Vohra (1998) points to a significant gain in productivity levels in the mining states until the end of the second oil price shock and a drastic fall thereafter. Although these results are based on US states, one can arguably conjecture that such changes may have had affected the major cities of these states. In this regard, our view of the TQPPP as a weaker version of the TPPP Balassa-Samuelson definition can be justified.[7] It is worth mentioning that this broader definition of the TQPPP concept nests the Balassa-Samuelson and Dornbusch and Vogelsang (1991) concepts of PPP.

Evidence in favor of QPPP or TQPPP does not imply that PPP as defined in Cassel (1918), Balassa (1964) and Samuelson (1964) is fulfilled, since in these cases PPP requires reversion towards a constant mean or a constant trend in the long-run. Therefore, in the presence of structural breaks, QPPP and/or TQPPP is necessary but not sufficient condition for the PPP to hold. In this case, when we have found evidence in favor of QPPP and/or TQPPP further investigations should be conducted to conclude that the PPP hypothesis is satisfied according to the definitions in Cassel (1918), Balassa (1964) and Samuelson (1964). To be specific, we require to impose the so-called parity restrictions on the coefficients of the first and last regimes so that the coefficients of these regimes are of the same magnitude. Note that after imposing the parity restrictions the deterministic component does not change in the long-run.

[7] Engel (1999) demonstrates that nearly all variability in real exchange rates against the United States can be attributed due to changes in the countries' relative consumer price of traded goods. Engel's (1999) result is a striking contradiction of the Balassa-Samuelson model, which necessities that all variability in real exchange rates is due to changes in international differences in two countries' relative price of traded to non-traded goods. In contrast, our results show support for the weak version of Balassa-Samuelson model within the US. This is to be expected since findings by both Parsley and Wei (1996) and Chen and Devereux (2003) indicate a much faster convergence in the relative consumer price of traded goods within the US.

This implies that after the last break has occurred, the deterministic component of the time series equals the one previous to the first structural break – see Papell (2002) and Papell and Prodan (2006). In this paper we consider all these cases, and propose a new way to accommodate for the presence of multiple structural breaks when testing for the different definitions of PPP that have been described.

3 Methodology

This section briefly discusses the panel stationarity tests proposed in Hadri (2000) and Carrion-i-Silvestre, del Barrio-Castro and López-Bazo (2005). These statistics are the ones applied in the paper to investigate the different definitions of PPP described in the previous section. This has led us to design a new procedure that allows us to test the PPP hypothesis with the inclusion of multiple structural breaks. Then, we briefly discuss about the effects of cross-section dependence when assessing the stochastic properties of panel data sets. Finally, we present two statistics to formally test the hypothesis of cross-section independence. All these statistics are used throughout the paper.

3.1 Panel stationarity tests with structural breaks

Hadri (2000) proposes a panel data stationarity test without structural breaks, while Carrion-i-Silvestre et al. (2005) extend the analysis to account for the presence of multiple structural breaks. Since the latter proposal encompasses the former one, we proceed to present the approach in Carrion-i-Silvestre et al. (2005). As above, let $y_{i,j,t} = (\ln p_{i,t} - \ln p_{j,t})$ be the difference between the logarithm of the price of two time series, for which we assume that its behavior can be modeled through:

$$y_{i,j,t} = \alpha_{i,j} + \sum_{k=1}^{m_{i,j}} \theta_{i,j,k} DU_{i,j,k,t} + \beta_{i,j} t + \sum_{k=1}^{m_{i,j}} \gamma_{i,j,k} DT^{*}_{i,j,k,t} + \varepsilon_{i,j,t}, \qquad (1)$$

where $t = 1, ..., T$ and $i, j = 1, ..., N$, $i \neq j$. The dummy variables $DU_{i,j,k,t}$ and $DT^{*}_{i,j,k,t}$ are defined as $DU_{i,j,k,t} = 1$ for $t > T^{i,j}_{b,k}$ and 0 elsewhere, and $DT^{*}_{i,k,t} = t - T^{i,j}_{b,k}$ for $t > T^{i,j}_{b,k}$ and 0 elsewhere, with $T^{i,j}_{b,k}$ denoting the k-th date of the break for the (i, j) pair of time series, $k = 1, ..., m_{i,j}, m_{i,j} \geq 1$, $\alpha_{i,j}$ and $\beta_{i,j}$ are the parameters of the constant and linear time trend, respectively, and $\varepsilon_{i,j,t}$ denotes the disturbance term. Note that the proposal in Hadri (2000) follows from setting $\theta_{i,j,k} = \gamma_{i,j,k} = 0$ $\forall i, j, k$, $i \neq j$, in (1).

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The model in (1) includes individual effects, individual structural break effects (i.e., shift in the mean caused by the structural breaks known as temporal effects where $\beta_{i,j} \neq 0$) and temporal structural break effects (i.e., shift in the individual time trend where $\gamma_{i,j} \neq 0$). In addition, the specification given by (1) considers multiple structural breaks, which are located at different unknown dates and where the number of breaks are allowed to vary across the members of the panel. Note that the different concepts of PPP that have been defined in the previous section appear as particular cases of the model in (1). Thus, for the (i, j)-th pair of time series the Cassel's definition of PPP is achieved when $\alpha_{i,j} \neq 0$ and $\beta_{i,j} = \theta_{i,j,k} = \gamma_{i,j,k} = 0$ $\forall k$ in (1), TPPP is obtained when $\alpha_{i,j} \neq \beta_{i,j} \neq 0$ and $\theta_{i,j,k} = \gamma_{i,j,k} = 0$ $\forall k$ in (1), QPPP is found when $\alpha_{i,j} \neq \theta_{i,j,k} \neq 0$ and $\beta_{i,j} = \gamma_{i,j,k} = 0$ $\forall k$ in (1) and, finally, TQPPP is found when $\alpha_{i,j} \neq \beta_{i,j} \neq \theta_{i,j,k} \neq \gamma_{i,j,k} \neq 0$ $\forall k$ in (1).

The test statistic is constructed by computing the stationarity test in Kwiatkowsky, Phillips, Schmidt and Shin (1992) – hereafter, KPSS test – for every member of the panel and then averaging the individual statistics. The general expression for the test statistic is:

$$LM(\lambda) = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \eta_{i,j}(\lambda_{i,j}), \tag{2}$$

with $\eta_{i,j}(\lambda_{i,j}) = \hat{\omega}_{i,j}^{-2} T^{-2} \sum_{t=1}^{T} \hat{S}_{i,j,t}^2$, where $\hat{S}_{i,j,t} = \sum_{l=1}^{t} \hat{\varepsilon}_{i,j,l}$ is the partial sum process that is obtained using the estimated OLS residuals of (1). $\hat{\omega}_{i,j}^2$ denotes a consistent estimate of the long-run variance of the error $\varepsilon_{i,j,t}$, which can be estimated following the procedure in Sul, Phillips and Choi (2005) – below we use this approach with the Quadratic spectral kernel. In (2), $\lambda_{i,j}$ is defined as the vector $\lambda_{i,j} = \left(\lambda_{i,j,1}, ..., \lambda_{i,j,m_{i,j}}\right)' = \left(T_{b,1}^{i,j}/T, ..., T_{b,m_{i,j}}^{i,j}/T\right)'$, which indicates the relative position of the date of the breaks on the entire time period, T, for each (i, j)-th pair of time series – note that for the test in Hadri (2000) $\lambda_{i,j} = 0$ $\forall i, j$, $i \neq j$, since there is no structural breaks. Assuming that time series are cross-section independent, Carrion-i-Silvestre et al. (2005) show that, under the null hypothesis of panel stationary with multiple shifts, the $LM(\lambda)$ statistic converges to:

$$Z(\lambda) = \frac{\sqrt{N}(LM(\lambda) - \bar{\xi})}{\bar{\varsigma}} \rightarrow N(0,1),$$

where $\bar{\xi}$ and $\bar{\varsigma}$ are the cross-sectional average of the individual mean and variance of $\eta_{i,j}(\lambda_{i,j})$, which are defined in Carrion-i-Silvestre et al. (2005).

In order to estimate the number of breaks and their locations, Carrion-i-Silvestre et al. (2005) follow the procedure developed by Bai and Perron (1998), which proceeds in two steps.[8] First, the breakpoints are estimated by globally minimizing the sum of squared residuals for all permissible values of $m_{i,j} \leq m^{\max}$, $i, j = 1, ..., N$, $i \neq j$. Second, we use the sequential testing procedure suggested in Bai and Perron (1998) to estimate the number of structural breaks. As a result, we obtain the estimation of both the number and position of the structural breaks. This procedure is then repeated $N(N-1)/2$ times to obtain the estimated number of breaks and their locations for each time series. It is worth mentioning that it would be possible to use the proposal in Qu and Perron (2007) to estimate multiple structural breaks in a system of equations.[9] However, to implement our proposal the approach in Bai and Perron (1998) suffices, since it only requires a consistent estimate of the number and position of the structural breaks. Finally, our approximation also considers the situation of no structural breaks, so that the case in which some time series might be not affected by the presence of structural breaks is taken into account.

As stated in Pesaran (2007a), the test outcomes across the different pairs are dependent – note that some pairs involve the same original price time series – although the proportion of pairs that meet the stationarity conditions can still be estimated consistently for N and T sufficiently large. Further, the fraction of the $N(N-1)/2$ real exchange rate pairs for which the null hypothesis is expected to be rejected, when it is true, equals the nominal size of the statistic – see Pesaran (2007a, p. 322). This implies that, asymptotically, the actual size of the panel data statistic equals the nominal one when we compute more than two pair-wise tests.

3.2 PPP hypothesis with structural breaks

The use of the individual KPSS and the panel stationarity statistic that have been described so far allow us to detect QPPP and/or TQPPP hypothesis when structural breaks are involved. Notwithstanding, evidence in favor of the

[8] Note that the sequential approach in Bai and Perron (1998) can be used here since under the null hypothesis of the statistic we have that the units are I(0) stationary processes. Therefore, the procedure in Bai and Perron (1998), which was proposed to estimate the number and position of the structural breaks in an I(0) framework, can be used. Under the alternative hypothesis of I(1) the test statistic is consistent even in the case where the break points are not properly estimated, as shown in Lee, Huang and Shin (1997), Kurozumi (2002) and Carrion-i-Silvestre (2003), among others.

[9] We thank an anonymous referee for pointing out this possibility.

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QPPP and/or TQPPP does not imply that PPP holds. If we are interested in testing the PPP hypothesis we should include the so-called parity restrictions in the model so that the parameters of the first regime are equal to the ones in the last regime. This has led us to extend the previous approach to consider these parity restrictions when there are multiple structural breaks.

Let us consider the data generating process (DGP) given by (1) expressed using orthogonal regressors for the model that includes a time trend with both level and slope shifts as:

$$y_{i,j} = x_{i,j}\delta_{i,j} + \varepsilon_{i,j}, \tag{3}$$

where $x_{i,j} = diag\left(x_{i,j,1}, \ldots, x_{i,j,m_{i,j}+1}\right)$, $x_{i,j,k,t} = (1, t)$, $\delta_{i,j} = \left(\delta_{i,j,1}, \ldots, \delta_{i,j,m_{i,j}+1}\right)'$ and $\delta_{i,j,k} = \left(\mu_{i,j,k}, \beta_{i,j,k}\right)'$ for $T^{i,j}_{b,k-1} < t \leq T^{i,j}_{b,k}$, $k = 1, \ldots, m_{i,j} + 1$, with the convention that $T^{i,j}_{b,0} = 0$ and $T^{i,j}_{b,m_{i,j}+1} = T$, being $m_{i,j}$ the number of structural breaks for the (i, j)-th pair of time series, $i, j = 1, \ldots, N$, $i \neq j$. The parity restrictions imply that the parameters of the first regime, $\delta_{i,j,1}$, and the ones for the last regime, $\delta_{i,j,m_{i,j}+1}$, have to be equal, while the parameters of the other regimes are left free. Note that this is the case because we have specified the model in a way that regressors are block orthogonal, so that the parameters on each orthogonal block are not affected by the values of the parameters in the previous blocks.[10] In this set-up these parity restrictions can be expressed as $R\delta_{i,j} = r$ with $R = \begin{bmatrix} I_l & 0_{l\times(m_{i,j}-1)l} & -I_l \end{bmatrix}$ and $r = 0_{(m_{i,j}+1)l\times 1}$, where I_l denotes the identity matrix, l is the number of regressors in $x_{i,j,k}$ – $l = 1$ in the constant only case and $l = 2$ in the case of the linear time trend – and $0_{a\times b}$ an $(a \times b)$-matrix of zeros. Using these elements, we can compute the restricted least squares estimator $\left(\hat{\delta}^*_{i,j}\right)$ of $\delta_{i,j}$ in (3) such that the estimator satisfies $R\delta_{i,j} = r$. It is worth mentioning that we require at least two structural breaks in order to impose the parity restrictions, since parity restrictions for the one break case will imply the absence of the structural break.

The estimation of the restricted least squares estimator of $\delta_{i,j}$ is carried out using the dynamic programming algorithm recently proposed in Perron and Qu (2006), which permits the consideration of multiple structural breaks with restrictions among the parameters of the different regimes. The proposal that we suggest in this paper proceeds in two stages: (i) we estimate the number of structural breaks using the unrestricted dynamic algorithm in Bai and Perron (1998) and, conditional to the number of structural breaks, (ii) minimize the

[10] This would not be the case if the model is not specified in terms of orthogonal regressors. Then, the restrictions on the parameters should be imposed so that the sum of all the coefficients for the level shifts (and the slope shifts if we deal with a trending variable) should be equal to zero.

restricted sum of squared residuals to estimate the position of the structural breaks with the vector of parameters $\delta^*_{i,j}$ satisfying $R\delta_{i,j} = r$, with R and r defined above.

The restricted estimated disturbance term $\hat{\varepsilon}^*_{i,j}$ can be used to compute the stationarity tests proposed in the previous section, i.e., the individual-by-individual restricted KPSS statistic given by:

$$\eta^*_{i,j}\left(\lambda_{i,j}\right) = \hat{\omega}^{-2}_{i,j} T^{-2} \sum_{t=1}^{T} \hat{S}^{*2}_{i,j,t}, \tag{4}$$

and the corresponding restricted panel data $(Z^*(\lambda))$ statistic. The following Theorem provides the limiting distribution of the individual KPSS with multiple structural breaks that considers the parity restrictions.

Theorem 1 *Let $\{y_{i,j,t}\}_{t=1}^{T}$ be the stochastic process generated according to (3), with $\{\varepsilon_{i,j,t}\}_{t=1}^{T}$ a stochastic process satisfying the strong mixing regularity conditions defined in Phillips and Perron (1988). Furthermore, let $\delta^*_{i,j}$ the vector of parameters that satisfies $R\delta_{i,j} = r$, with $R = \left[I_l \quad 0_{l\times(m_{i,j}-1)l} \quad -I_l\right]$, where $l = 1$ for the model that includes a constant and level shifts, and $l = 2$ for the model that includes a linear time trend with both level and slope shifts. Thus, as $T \to \infty$ and $T^{i,j}_{b,k} \to \infty$ in a way that $\lambda_{i,j,k} = T^{i,j}_{b,k}/T\ \forall k,\ k = 1,\ldots,m_{i,j}$, remains constant, the $\eta^*_{i,j}\left(\lambda_{i,j}\right)$ statistic given by (4) converges to:*

$$\eta^*_{i,j}\left(\lambda_{i,j}\right) \Rightarrow \int_0^{\lambda_{i,j,1}} M_{i,j,1}\left(\lambda_{i,j}\right)^2 dr + \cdots + \int_{\lambda_{i,j,m_{i,j}}}^{1} M_{i,j,m_{i,j}+1}\left(\lambda_{i,j}\right)^2 dr,$$

where $\Rightarrow$ indicates weak convergence to the associated measure of probability and $M_{i,j,k}\left(\lambda_{i,j}\right)$ denotes the orthogonal projection of a standard Brownian motion onto the space spanned by the regressors and the terms involved in the definition of the restrictions.

The proof of the Theorem is given in Appendix. The limit distribution of the statistic depends on the number of structural breaks as well as on their relative position in the sample. We have computed asymptotic critical values for $m = 2,\ldots,9$ structural breaks, for all possible combinations of $\lambda_k = \{0.1, 0.2, \ldots, 0.9\}$, $k = 1,\ldots,m$. In order to summarize the information, we have estimated response surfaces to approximate the asymptotic critical values, for which we have essayed the following functional form:

$$cv(\lambda) = \beta_{0,0} + \beta_{0,1}m + \sum_{k=1}^{9} \beta_{1,k}\lambda_k + \sum_{l=1}^{3}\sum_{k=1}^{7}\sum_{j=k+1}^{8} \delta_{l,k,j} \left|\lambda_k - \lambda_j\right|^l + u,$$

where λ is a (9×1)-vector of the sorted (in ascending order) values of the break fraction parameters.[11] In addition, we have used the same functional form to approximate the mean and the variance of the statistics for each combination of break fractions, which is required in the computation of panel data statistics similar to those defined above for the non-restricted case. We do not report here the estimated coefficients of these response surfaces to save space, although they are available from the authors upon request.[12]

3.3 Testing for cross-section independence

Recent developments offer the possibility of testing for the presence of cross-section dependence among time series. Pesaran (2004) designs a test statistic based on the average of pair-wise Pearson's correlation coefficients $\hat{p}_j$, $j = 1, 2, \ldots, n$, $n = N(N-1)/2$, of the residuals obtained from an autoregressive (AR) model that include dummy variables to capture the structural breaks. The CD statistic in Pesaran (2004) is given by

$$CD = \sqrt{\frac{2T}{n}} \sum_{j=1}^{n} \hat{p}_j \to N(0,1).$$

This statistic tests the null hypothesis of cross-section independence against the alternative of dependence.

Besides, Ng (2006) relies on the computation of spacings to test the null hypothesis of independence. In brief, the procedure in Ng (2006) works as follows. First, we get rid of autocorrelation pattern in individual time series through the estimation of an AR model. As for the test in Pesaran (2004), this allows us isolating cross-section regression from serial correlation. Taking the estimated residuals from AR models with dummy variables as individual series, we compute the absolute value of Pearson's correlation coefficients ($\bar{p}_j = |\hat{p}_j|$) for all possible pairs of time series, $j = 1, 2, \ldots, n$, where as above

[11] When there are less than nine structural breaks ($m_{i,j} < 9$), the first $m_{i,j}$ positions of the λ vector collect the break fractions and the other $(9 - m_{i,j})$ positions are zero.

[12] The GAUSS procedure that implements the estimated response surfaces can be downloaded at http://www.syedbasher.org/ or http://www.eco.ub.es/~carrion/.

$n = N(N-1)/2$, and sort them in ascending order. As a result, we obtain the sequence of ordered statistics given by $\{\bar{p}_{[1:n]}, \bar{p}_{[2:n]}, \ldots, \bar{p}_{[n:n]}\}$. Under the null hypothesis that $p_j = 0$ and assuming that individual time series are Normal distributed, $\bar{p}_j$ is half-normally distributed. Furthermore, let us define $\bar{\phi}_j$ as $\Phi\left(\sqrt{T}\bar{p}_{[j:n]}\right)$, where Φ denotes the cdf of the standard Normal distribution, so that $\bar{\phi} = (\bar{\phi}_1, \ldots, \bar{\phi}_n)$. Finally, let us define the spacings as $\Delta\bar{\phi}_j = \bar{\phi}_j - \bar{\phi}_{j-1}$, $j = 1, \ldots, n$.

Second, Ng (2006) proposes splitting the sample of (ordered) spacings at arbitrary $\vartheta \in (0,1)$, so that we can define the group of small (S) correlation coefficients and the group of large (L) correlation coefficients. The definition of the partition is carried out through minimization of the sum of squared residuals

$$Q_n(\vartheta) = \sum_{j=1}^{[\vartheta n]} \left(\Delta\bar{\phi}_j - \bar{\Delta}_S(\vartheta)\right)^2 + \sum_{j=[\vartheta n]+1}^{n} \left(\Delta\bar{\phi}_j - \bar{\Delta}_L(\vartheta)\right)^2,$$

where $\bar{\Delta}_S(\vartheta)$ and $\bar{\Delta}_L(\vartheta)$ denotes the mean of the spacings for each group respectively. Consistent estimate of the break point is obtained as $\vartheta \hat{=} \arg\min_{\vartheta\in(0,1)} Q_n(\vartheta)$, where definition of some trimming is required – we follow Ng (2006) and set trimming at 0.10.

Once the sample has been split, we can proceed to test the null hypothesis of non-correlation in both sub samples. Obviously, rejection of the null hypothesis for the small correlations sample will imply rejection for the large correlations sample provided that the statistics are sorted in ascending order. Therefore, the null hypothesis can be tested for the small (S), large (L) and the whole (W) sample using the Spacing Variance Ratio $SVR(\eta)$ in Ng (2006), with $\hat{\eta} = \left[\hat{\vartheta}n\right]$ being the number of statistics in the small correlations group. Ng (2006) shows that under the null hypothesis that a subset of correlations is jointly zero, the standardized statistic $svr(\eta) \to N(0,1)$.

One advantage of the approach in Ng (2006) is that it allows us gaining some insight about the kind of cross-section dependence in terms of how pervasive and strong is the cross-section correlation. The use of these statistics will help us to decide in which panel stationarity statistic we should most base the statistical inference.

3.4 Cross-section dependence

The presentation of the panel statistics so far has assumed that time series are cross-section independent. However, this assumption might be restrictive in practice since the analysis of macroeconomic time series for different countries are affected by similar major events that might introduce dependence among time series in the panel data set.[13] There are different approximations in the literature to deal with cross-section dependence. In this paper we account for cross-section dependence in two ways. First, we follow the suggestion in Levin, Lin and Chu (2002) and proceed to remove the cross-section mean, which is equivalent to include temporal effects in the panel data set. Second, we follow Maddala and Wu (1999) and compute the empirical distribution by means of parametric bootstrap. These two approaches are applied for all test statistics described above.

Other proposals in the literature that deal with cross-section dependence are O'Connell (1998), who estimates a SUR specification, and Bai and Ng (2004a, b), Moon and Perron (2004), Harris, Leybourne and McCabe (2005) and Pesaran (2007b), who use common factor models. However, in our case the statistic in Ng (2006) that is computed below indicates that the presence of cross-section dependence is not pervasive, so that a common factor structure does not appear to be a suitable characterization of the cross-section dependence in the sample that is used.

4 Finite sample performance

We have conducted a small set of Monte Carlo simulations to assess the finite sample performance of the restricted panel data statistic. The DGP is given by:

$$y_{i,j,t} = \alpha_{i,j} + \sum_{k=1}^{m_{i,j}} \theta_{i,j,k} DU_{i,j,k,t} + \beta_{i,j} t + \sum_{k=1}^{m_{i,j}} \gamma_{i,j,k} DT^{*}_{i,j,k,t} + \varepsilon_{i,j,t} + \varphi_{i,j,t}, \quad (5)$$

with $\alpha_{i,j} = \beta_{i,j} = 0 \ \forall i, j, \ \theta_{i,j,k} \sim U(4, 5), \ \gamma_{i,j,k} \sim U(0.4, 0.5)$, with two structural breaks ($m_{i,j} = 2 \ \forall i, j$), where U denotes the uniform distribution. The disturbance term $\varepsilon_{i,j,t}$ is generated according to $\varepsilon_{i,j,t} = \rho_{i,j}\varepsilon_{i,j,t-1} + v_{i,j,t}$

[13] O'Connell (1998) shows the importance of cross-section dependence in PPP analyses. Recently, using simulation methods, Banerjee, Marcellino and Osbat (2004) show that panel data unit root statistics tend to conclude in favor of stationarity, or convergence, when cross-section dependence is not considered.

using two sets of values for the autoregressive parameter: (i) $\rho_{i,j} \sim U(0.5, 0.6)$ and (ii) $\rho_{i,j} = 0.8 \ \forall i, j$. Further, we consider independent and cross-section dependent time series through the definition of $v_{i,j,t}$. First, the cross-section independent case defines $v_{i,j,t} \sim iid\, N(0, 1)$. Second, the effects of cross-section dependence have been studied using $v_t \sim N(0, \Omega_v)$, with the covariance matrix Ω_v defined following the same steps as in Chang (2002):

1. Generate an $(N \times N)$ matrix $\Sigma_v \sim U(0, 1)$.
2. Construct from Σ_v an orthogonal matrix $H = \Sigma_v (\Sigma_v' \Sigma_v)^{-1/2}$.
3. Generate a set of N eigenvalues, $\varrho_1, \varrho_2, \ldots, \varrho_N$, with $\varrho_1 = 0.1$, $\varrho_N = 1$ and draw $\varrho_2, \ldots, \varrho_{N-1}$ from a $U(0.1, 1)$.
4. Form a diagonal matrix Υ with $\varrho_1, \varrho_2, \ldots, \varrho_N$ on the diagonal.
5. Construct the covariance matrix Ω_v as $\Omega_v = H \Upsilon H'$.

The empirical size of the statistics is analyzed specifying $\varphi_{i,j,t} = 0 \ \forall i, j, t$ in (5), whereas for the empirical power we set:

$$\varphi_{i,j,t} = \varphi_{i,j,t-1} + w_{i,j,t},$$

where $w_{i,j,t} \sim iid\, N(0, \sigma_w^2)$ with three different values for $\sigma_w^2 = \{0.001, 0.01, 0.1\}$. Note that for the cross-section independent case, σ_w^2 has the interpretation as the signal-to-noise ratio since the variance of $v_{i,j,t}$ equals one – this interpretation cannot be done for the cross-section dependent case because the variance of $v_{i,j,t}$ does not necessarily equal one. The simulations are carried out for all combinations of $T = \{100, 200, 300\}$ and $N = \{20, 40\}$ considering both the QPPP and TQPPP specifications. The long-run variance has been estimated as described above, using the approach in Sul et al. (2005) with the Quadratic spectral kernel.

Table 1 reports the empirical size of the statistic for the cross-section independent (Panel A) and cross-section dependent (Panel B) time series. For the independent case, we can see that the empirical size achieves the nominal 5% level as T increases for the QPPP specification, but the test is mildly undersized for the TQPPP specification. As expected, the introduction of unattended cross-section dependence leads to huge size distortions in the test in all cases – larger size distortions are observed for the TQPPP specification. Therefore, cross-section dependence has to be taken into account in empirical analyses if size distortions are to be avoided.

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Table 2 presents the empirical power. Let us first focus on the results that are based on $\rho_{i,j} \sim U(0.5, 0.6)$. For the cross-section independent case (Panel A in Table 2), we see that for small values of the signal-to-noise ratio (σ_w^2), higher empirical power is obtained for the QPPP specification than for the TQPPP one. Note that the lower power shown by the TQPPP compared to the QPPP is due to the larger number of parameters involved in the former specification. However, the empirical power approaches one as σ_w^2, T and N increase. The picture changes when we analyse the effects of unattended cross-section dependence – Panel B in Table 2. In this case the empirical power seems to be stagnated at similar values regardless of the magnitude of σ_w^2 and, although slightly, it decreases as T increases. However, due to the large size distortions that the statistics have shown in this situation, the empirical power analysis in this case is meaningless.

In contrast, when the autoregressive parameter changes to $\rho_{i,j} = 0.8 \; \forall i, j$ the empirical power decreases compared to previous results. If we focus on the cross-section independent time series case – see Panel C in Table 2 – we can observe that the power of the panel data statistic is higher for the QPPP specification than for the TQPPP one. As before for both specifications the empirical power increases with σ_w^2, T and N, reaching one for the combinations with large σ_w^2, T and N. When cross-section dependence is introduced, the empirical power seems to be stagnated around the same values regardless of σ_w^2 – see Panel D in Table 2. As mentioned above, the size distortion that is found when cross-section dependence is not accounted for, makes the empirical power analysis to be meaningless.

In all, when the requirements of the model specification are met, the empirical size of the statistics that we have proposed is close to the nominal one, and they have non-negligible empirical power. Thus, we can see that, for the cross-section independent case, the empirical size is close to the nominal one – especially for large T – and the presence of structural breaks does not cause power loses. The test statistics that we propose suffer from size distortions when unattended cross-section dependence is introduced, although this is to be expected provided that our framework assumes the time series to be cross-section independent. This shows the importance to consider cross-section dependence (when it is present) to obtain correct conclusions from the statistical inference. In the empirical analysis below we deal with cross-section dependence using bootstrapped critical values.

5 Empirical results

5.1 Data

We extend the data set used by Cecchetti et al. (2002) and Chen and Devereux (2003) to include more recent observations consisting annual Consumer Price Index (CPI) covering the period 1918 to 2005 ($T = 88$) for $N = 17$ US cities: Atlanta, Boston, Chicago, Cincinnati, Cleveland, Detroit, Houston, Kansas City, Los Angeles, Minneapolis, New York, Philadelphia, Pittsburgh, Portland, San Francisco, Seattle, and St. Louis.[14] All data were extracted from the Bureau of Labor Statistics's (BLS) webpage (www.bls.gov).

The limitation of the BLS price indices is that they do not measure absolute city price levels and, hence, they do not provide information on the relative cost of living across cities at a point in time. In order to overcome this drawback, we follow Chen and Devereux (2003) to compute price level series. In brief, Chen and Devereux (2003) use the absolute cost of living prices that Koo, Phillips and Sigalla (2000) obtained using disaggregated BLS data for 1989. Thus, we take the absolute price level for 1989 in Koo et al. (2000) as the starting point and apply (backward and forward in time) the inflation rates calculated from individual city price indices to obtain absolute price indices for other years. Chen and Devereux (2003) also check the reliability of their estimates on the basis of two other benchmark years (1935 and 1975) and concluded that their proxy for price dispersion is close to alternative benchmarks.

We only dispose of information for general price level but not disaggregate price level since we do not have the baskets for different concepts. Therefore, we maintain the analysis at a general aggregated level. This may explain why the presence of a time trend is required since, unlike individual price levels, aggregate price levels may not adjust so quickly due to differences in productivity among traded and non-traded sectors. Thus, when the absolute city prices are converging, bilateral real exchange rates may contain a time trend with or without structural breaks depending on the pattern of convergence. In fact, Chen and Devereux (2003) graphically identified the evidence of structural breaks and time trends in the US city real exchange rate for the period 1918-2000. However, there might be more than one structural break affecting the real exchange rates so that the analysis should cover general situations in which both the number and location of the structural breaks are unknown.

[14] Note that, the original Cecchetti et al. (2002)'s sample consists of 19 cities including Baltimore and Washington DC. However, since 1996, the Bureau of Labor Statistics no longer maintains separate data for these two cities. As a result, these cities are excluded from the analysis.

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5.2 Price convergence and structural breaks

Robust conclusions to the specification of the benchmark can be obtained if we base the study on all possible pair of exchange rates, i.e., $N(N-1)/2 = 136$ pairs in this case. As mentioned above, the number of break points is estimated following the procedure in Bai and Perron (1998) with the sequential testing or the LWZ information criterion in Liu, Wu and Zidek (1998) depending on the presence of broken linear trends. The initial maximum number of structural breaks is $m^{\max} = 5$, although in few cases the maximum was achieved for which $m^{\max}$ has been increased to $m^{\max} = 8$ – the new maximum was never reached.

Table 3 summarizes the proportion of rejections of the null hypothesis of I(0) in each case – detailed results are available from the authors in a companion appendix. We can see that more evidence against the null hypothesis is found when the TQPPP specification is considered. Thus, using 5% critical values the null hypothesis is rejected for 13.2% of all possible pairs when using the QPPP specification, while the proportion increases up to the 35.3% for the TQPPP one. We have reported the proportion of rejections that is obtained when the BIC information criterion is used to select between the QPPP and the TQPPP hypothesis specifications for each time series (mixed case). In this situation, the proportion of rejections is 33.8% using the critical values at the 5% level of significance. Therefore, we cannot conclude that there is strong evidence against the null hypothesis of either QPPP, TQPPP or the mixture QPPP/TQPPP versions of the PPP hypothesis.

This individual information can be combined to define panel data statistics. Before proceeding, we have computed the statistics in Pesaran (2004) and Ng (2006) to test the null hypothesis of cross-section independence. The CD statistic for the constant with level shifts equals 27.113, while for the linear trend with level and slope shifts is 23.66. As can be seen, for both deterministic specifications the null hypothesis of independence is strongly rejected. The same conclusion is found when we compute the statistic in Ng (2006). Thus, for the constant with level shifts we obtain for the whole sample $svr^W(\hat{\eta}) = 26.610$ (p-value 0.000), for the large sample $svr^L(\hat{\eta}) = 23.935$ (p-value 0.000) and for the small sample $svr^S(\hat{\eta}) = 2.260$ (p-value 0.012), which indicate strong rejection of the null hypothesis of cross-section independence. In this case, the proportion of individuals in the small sample is 0.503. For the deterministic component given by the linear trend with level and slope shifts we have $svr^W(\hat{\eta}) = 23.657$ (p-value 0.000), $svr^L(\hat{\eta}) = 21.714$ (p-value 0.000) and $svr^S(\hat{\eta}) = 2.786$ (p-value 0.003), with the proportion of individuals in the small sample that equals 0.58. From these results we can conclude that cross-section is present among the individuals that define the panel data set, although this dependence is not strong, provided that half of the correlations are located in the small sample.

The results on the computation of the panel data statistics are reported in Table 4. When cross-section dependence is taken into account through cross-section demeaning, the null hypothesis of I(0) cannot be rejected for the QPPP specification at the 5% level of significance, although the null hypothesis is clearly rejected for the TQPPP and the mixture of the QPPP/TQPPP specifications.

Conclusive results are obtained if we base the inference on the bootstrapped critical values. In this case, the null hypothesis of I(0) is not rejected at the 5% level for either the QPPP, the TQPPP or the mixture of the QPPP/TQPPP hypotheses regardless of the statistic that is used.[15]

Table 5 lists, for both QPPP and TQPPP hypotheses, the number and position of the structural breaks for all possible pairs of a representative city, Atlanta. To save on space, we do not report the break points for all pairs of cities, but these results are available in a companion appendix.[16] For the QPPP hypothesis, the maximum number of breaks that has been estimated is six, while it is four for the TQPPP hypothesis. While there is no visible pattern concerning the number or timing of the break points, these results evidence the non-linearity of the real exchange rates among US cities. This requires to accommodate multiple structural breaks in the empirical analysis.

5.3 PPP and structural breaks

The consideration of the parity restrictions in the analysis provides the results reported in Table 3, where proportions of rejections of the null hypothesis of I(0) are presented. First of all, we can see that restricting the parameters of the QPPP model gives a proportion of rejections of 31.5% using the critical values at the 5% level of significance, while the proportion for the TQPPP hypothesis is of the 8.8%. These results show that imposing parity restrictions on the QPPP model specification may imply incredible restrictions that are not to be satisfied in practice. However, the converse is found for the TQPPP model specification, which shows that the more flexible specification that defines the TQPPP hypothesis is more likely to satisfy the parity restrictions. As

[15] As above, the selection of the model that is used to compute the panel data statistics with the mixture of the QPPP and TQPPP specifications is based on the BIC information criterion. Only for eighteen out of the one hundred and thirty-six pairs of cities the QPPP specification is selected.

[16] The companion appendix can be found in authors webpages.

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expected, the use of the BIC information criterion to select between these two specifications for each individual gives a proportion that lies between both situations – the QPPP specification is selected in the 23.8% of the cases.

Table 3 also reports the proportions of rejection for the non-restricted model that are obtained for the same time series for which the parity restrictions are of application. We can see that the proportion for the QPPP hypothesis is smaller (13%) when we do not impose the parity restrictions, while it increases for the TQPPP hypothesis (39.8%) and the mixed QPPP/TQPPP hypothesis (40.3%). These results indicate that more evidence is found in favor of the TPPP hypothesis than for the PPP hypothesis in those cases for which parity restrictions can be imposed.

The picture based on the individual statistics can be completed with the results from the panel data statistics. Table 4 indicates that the null hypothesis of I(0) is rejected at the 5% level of significance for both the restricted QPPP and TQPPP hypotheses when using the bootstrap critical values. These results show that there is no evidence in favor of the PPP hypothesis when we consider the whole panel data set. Therefore, we have to conclude that the PPP hypothesis does not hold for all the pairs of real exchange rates that have been considered in this paper.

Finally, Table 6 lists, for both QPPP and TQPPP hypotheses, the number and position of the structural breaks that have been estimated for Atlanta. It is worth recalling that at least two structural breaks are needed to impose the parity restriction, which explains why the QPPP or TQPPP hypotheses could not be tested for selected pairs of cities. Notice that, while imposing the parity restriction might imply a change in the estimated position of the structural breaks, the number of breaks remains identical with or without the parity restriction.

6 Conclusions

The main objective of this paper is to bring new light on the question of price level convergence and how it relates to PPP hypothesis. We suggest a new panel data based procedure for testing PPP, which is robust to multiple structural breaks and cross-sectional dependence.

A well-known complication associated with panel data framework is the way in which test outcomes are often interpreted in the literature. Thus, the null hypothesis in most of the existing panel stationarity and unit root test statistics implies a joint hypothesis that *all* of time series in the panel data set – the real exchange rates in our case – under consideration are realizations of either I(0) – for the stationarity tests – or I(1) nonstationary – for the unit root tests – processes. Therefore, a rejection of the null hypothesis only implies that there is *a significant proportion* of time series for which the null hypothesis is not satisfied. In this paper, we show that by resorting to the *pairwise* approach of PPP tests one can overcome the problem of dealing with incorrect interpretation of the null hypothesis in the context of panel data tests. The pairwise approach offers a natural way to estimate the proportion of the pairs of real exchange rates in the sample that satisfy the PPP hypothesis. As it stands, the magnitude of rejections of the null hypothesis is of more economic interest than otherwise.

Nevertheless, further complications arise in terms of alternative concepts of PPP when the data is characterized by the presence of multiple structural breaks. The primary contribution of this paper has been to develop a framework that can handle differing concepts of PPP while simultaneously providing the magnitude of the proportion of pairs that satisfy the PPP hypothesis.

As an empirical application, we have utilized aggregate annual price data from 1918 to 2005 for seventeen US cities. We find evidence for stationarity around a changing level when the parity restriction is not imposed, while imposing parity restriction provides favorable evidence for the specification that accounts for changes in the slope of the trend. When choosing between these specifications, more favorable evidence is found in favor of the TPPP hypothesis thus corroborating the Balassa-Samuelson version of PPP.

Our analysis does not shed light on the questions on (i) what determines the deviation from PPP and (ii) how long these deviations are likely to persist. Concerning the first question, Engel and Rogers (1996) show that both distance and border matter for relative price variability, whereas using a shorter data set than ours Cecchetti et al. (2002) do not find distance as a significant factor for real exchange rate adjustment.[17] The empirical literature on the second question is vast and is cogently summarized in Rogoff (1996). Treating half-life as a measure of persistence of deviation from parity, the consensus range of estimates of half-lives lie between three and five years. For the US city prices in question, Cecchetti et al. (2002) find a half-life of convergence of nearly nine years, well above the consensus range. A promising avenue for future work involves further investigation on the speed of convergence (i.e., half-life) for US city prices using the framework developed in this paper.

Finally, it is worth mentioning that our framework can be used in other

[17] Cecchetti et al. (2002) further document that transportation costs or the presence of non-traded goods in price indices account only fragmentary evidence for the slow adjustment of the overall consumer price index.

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fields of economics as well. For instance, the approach can be used in empirical applications that analyze interest rate parity, convergence in wages, and Fisher effect, among others. We expect that these and related applications will be exciting avenues for future research.

A Mathematical appendix

Throughout the Appendix and unless strictly necessary, we avoid the use of the i and j subscripts that have been used to denote the (i,j)-th pair of time series to simplify the notation.

Lemma 1 *Let us define the x_k $((T_k - T_{k-1}) \times 2)$-matrix defined with the row vector $x_{k,t} = (1, t)$, $T_{k-1} < t \leq T_k$, $t = 1, \ldots, T$, $k = 1, \ldots, m+1$, with $T_0 = 0$ and $T_{m+1} = T$. Let $P_k = diag\left(T^{-1/2}, T^{-3/2}\right)$ be a scaling matrix, and $\{\varepsilon_t\}_{t=1}^T$ be a stochastic process satisfying the strong mixing regularity conditions defined in Phillips and Perron (1988). Then, as $T \to \infty$:*

$$\text{(a) } P_k x_k' x_k P_k \to \begin{bmatrix} (\lambda_k - \lambda_{k-1}) & \frac{1}{2}\left(\lambda_k^2 - \lambda_{k-1}^2\right) \\ \frac{1}{2}\left(\lambda_k^2 - \lambda_{k-1}^2\right) & \frac{1}{3}\left(\lambda_k^3 - \lambda_{k-1}^3\right) \end{bmatrix} \equiv H_k\left(\lambda_{k-1}, \lambda_k\right)$$

$$\text{(b) } P_k x_k' \varepsilon \Rightarrow \left(\sigma\left(W\left(\lambda_k\right) - W\left(\lambda_{k-1}\right)\right), \sigma \int_{\lambda_{k-1}}^{\lambda_k} r dW\left(r\right)\right)' \equiv \omega L_k\left(\lambda_{k-1}, \lambda_k\right),$$

where $\lambda_k = T_k/T$, with $\lambda_0 = 0$ and $\lambda_1 = 1$, $r = t/T$, and $\omega^2 = \lim_{T\to\infty} T^{-1} E\left(S_T^2\right)$ with $S_t = \sum_{j=1}^t \varepsilon_j$.

Proof. Statement (a) in Lemma 1 follows from direct calculation, whereas statement (b) follows from the application of the Donsker's Theorem and the Continuous Mapping Theorem (CMT) – see Billingsley (1968). ■

A.1 Proof of Theorem 1

Without loss of generality, let us consider the situation in which we have two structural breaks that affect both the level and the slope of a linear time trend. The model that uses the deterministic specification that is given by a constant term and level shifts is obtained as a particular case. The derivations are valid for the case of multiple structural breaks. Note that we can write the model using orthogonal regressors so that the matrix of regressors x is block-diagonal

$$x = \begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_{m+1} \end{bmatrix} = diag\left(x_1, \ldots, x_{m+1}\right),$$

where $x_{k,t} = (1, t)$ for $T_{k-1} < t \leq T_k$, where $k = 1, \dots, m+1$, with the convention that $T_0 = 0$ and $T_{m+1} = T$, being m the number of structural breaks. Similarly, we can define the block-diagonal scaling matrix $P = diag\left(P_1, \dots, P_{m+1}\right)$, with $P_k = diag\left(T^{-1/2}, T^{-3/2}\right)$ $\forall k$, $k = 1, \dots, m+1$. The restricted estimated residuals $\left(\hat{\varepsilon}_t^*\right)$ are computed from

$$\hat{\varepsilon}_t^* = \varepsilon_t - x_t\left(\hat{\delta}^* - \delta\right), \tag{6}$$

where $\hat{\delta}^*$ denotes the restricted least squares estimator that satisfies the restriction given by $R\delta = r$. It can be shown that – see Judge et al. (1985) pp. 238:

$$\begin{aligned}\left(\hat{\delta}^* - \delta\right) &= \left(x'x\right)^{-1} x'\varepsilon + \left(x'x\right)^{-1} R'\left[R\left(x'x\right)^{-1} R'\right]^{-1} \times \\ &\quad \left(r - R\beta - R\left(x'x\right)^{-1} x'\varepsilon\right) \\ &= \left(x'x\right)^{-1} x'\varepsilon - \left(x'x\right)^{-1} R'\left[R\left(x'x\right)^{-1} R'\right]^{-1} R\left(x'x\right)^{-1} x'\varepsilon, \end{aligned} \tag{7}$$

where in our case the matrix that defines the parameter restrictions is given by the $(l \times (m+1)\, l)$-matrix $R = \left[I_l \; 0_{k\times(m-1)l} \; -I_l\right]$, where l is the number of regressors in each subperiod, i.e., $l = 2$ in this case.

Let us first analyze the second element on the right hand side of (7), $A = \left(x'x\right)^{-1} R'\left[R\left(x'x\right)^{-1} R'\right]^{-1} R\left(x'x\right)^{-1} x'\varepsilon$. Note that we can scale the different elements of this term

$$\begin{aligned} A &= P\left(Px'xP\right)^{-1} R'\left[RP\left(Px'xP\right)^{-1} PR'\right]^{-1} RP\left(Px'xP\right)^{-1} Px'\varepsilon \\ &= A_1 A_2^{-1} A_3, \end{aligned}$$

so that the element given by $A_1 = P\left(Px'xP\right)^{-1} R'$ is

$$\begin{aligned} A_1 &= \begin{bmatrix} P_1\left(P_1 x_1' x_1 P_1\right)^{-1} P_1 & & 0 \\ & \ddots & \\ 0 & & P_{m+1}\left(P_{m+1} x_{m+1}' x_{m+1} P_{m+1}\right)^{-1} P_{m+1} \end{bmatrix} \times \\ &\quad \begin{bmatrix} I_l \\ 0_{(m-1)l\times l} \\ -I_l \end{bmatrix} \\ &= \begin{bmatrix} P_1\left(P_1 x_1' x_1 P_1\right)^{-1} P_1 \\ 0_{(m-1)l\times l} \\ -P_{m+1}\left(P_{m+1} x_{m+1}' x_{m+1} P_{m+1}\right)^{-1} P_{m+1} \end{bmatrix}. \end{aligned}$$

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The term $A_2 = RP\left(Px'xP\right)^{-1}PR'$ can be written as

$$\begin{aligned}
A_2 &= \begin{bmatrix} I_l \\ 0_{(m-1)l\times l} \\ -I_l \end{bmatrix}' \times \\
&\quad \begin{bmatrix} P_1\left(P_1x_1'x_1P_1\right)^{-1}P_1 & & 0 \\ & \ddots & \\ 0 & & P_{m+1}\left(P_{m+1}x_{m+1}'x_{m+1}P_{m+1}\right)^{-1}P_{m+1} \end{bmatrix} \times \\
&\quad \begin{bmatrix} I_l \\ 0_{(m-1)l\times l} \\ -I_l \end{bmatrix} \\
&= P_1\left[\left(P_1x_1'x_1P_1\right)^{-1} + \left(P_{m+1}x_{m+1}'x_{m+1}P_{m+1}\right)^{-1}\right]P_1,
\end{aligned}$$

provided that $P_1 = \ldots = P_k = \ldots = P_{m+1}$. Finally,

$$\begin{aligned}
A_3 &= \begin{bmatrix} I_l & 0_{l\times(m-1)l} & -I_l \end{bmatrix} \begin{bmatrix} P_1\left(P_1x_1'x_1P_1\right)^{-1}P_1x_1'\varepsilon \\ \vdots \\ P_{m+1}\left(P_{m+1}x_{m+1}'x_{m+1}P_{m+1}\right)^{-1}P_{m+1}x_{m+1}'\varepsilon \end{bmatrix} \\
&= P_1\left[\left(P_1x_1'x_1P_1\right)^{-1}P_1x_1'\varepsilon - \left(P_{m+1}x_{m+1}'x_{m+1}P_{m+1}\right)^{-1}P_{m+1}x_{m+1}'\varepsilon\right].
\end{aligned}$$

Using these element we can see that

$$\begin{aligned}
A &= A_1P_1^{-1}\left[\left(P_1x_1'x_1P_1\right)^{-1} + \left(P_{m+1}x_{m+1}'x_{m+1}P_{m+1}\right)^{-1}\right]^{-1} \\
&\quad \times \left[\left(P_1x_1'x_1P_1\right)^{-1}P_1x_1'\varepsilon - \left(P_{m+1}x_{m+1}'x_{m+1}P_{m+1}\right)^{-1}P_{m+1}x_{m+1}'\varepsilon\right] \\
&= A_1P_1^{-1}O_p(1),
\end{aligned}$$

provided that, from Lemma 1, $P_kx_k'x_kP_k = O(1)$ and $P_kx_k'\varepsilon = O_p(1)$, $k = 1,\ldots,m+1$.

The scaled restricted partial sum process $\hat{S}_t^* = T^{-1/2}\sum_{j=1}^t \hat{\varepsilon}_j^*$ defined using residuals in (6) is given by

$$\hat{S}_t^* = T^{-1/2}\sum_{j=1}^t \varepsilon_j - T^{-1/2}\sum_{j=1}^t x_jP\left(Px'xP\right)^{-1}Px'\varepsilon + T^{-1/2}\sum_{j=1}^t x_jA_1P_1^{-1}O_p(1).$$

Note that if we define $x_t = (x_{1,t},\ldots,x_{m+1,t})$, for a given $T_{k-1} < t \leq T_k$ we have

$$\begin{aligned} T^{-1/2}\sum_{j=1}^{t} x_j P_k &\rightarrow \left(\left(r-\lambda_{k-1}\right), \frac{1}{2}\left(r^2-\lambda_{k-1}^2\right)\right) \\ &\equiv x\left(r,\lambda_{k-1}\right), \end{aligned}$$

and

$$\begin{aligned} T^{-1/2}\sum_{j=1}^{t} x_j A_1 P_1^{-1} &= T^{-1/2}\sum_{j=1}^{t} x_{1,j} P_1 \left(P_1 x_1' x_1 P_1\right)^{-1} \\ &\quad -T^{-1/2}\sum_{j=1}^{t} x_{m+1,j} P_{m+1}\left(P_{m+1} x_{m+1}' x_{m+1} P_{m+1}\right)^{-1} \\ &= O\left(1\right). \end{aligned}$$

Using all these elements we can see that for the first segment, i.e., when $T_0 < t \leq T_1$, the process $\sigma^{-1}\hat{S}_t^*$ converges to

$$\begin{aligned} \hat{\omega}^{-1}\hat{S}_t^* &\Rightarrow W\left(r\right) - \left(x\left(r,\lambda_0\right), 0_{1\times ml}\right) H\left(\lambda\right)^{-1} L\left(\lambda\right) \\ &\quad +x\left(r,\lambda_0\right) H_1\left(\lambda_0,\lambda_1\right)^{-1} \times \\ &\quad \left[H_1\left(\lambda_0,\lambda_1\right)^{-1} + H_{m+1}\left(\lambda_m,\lambda_{m+1}\right)^{-1}\right]^{-1} \times \\ &\quad \left[H_1\left(\lambda_0,\lambda_1\right)^{-1} L_1\left(\lambda_0,\lambda_1\right) - H_{m+1}\left(\lambda_m,\lambda_{m+1}\right)^{-1} L_{m+1}\left(\lambda_m,\lambda_{m+1}\right)\right] \\ &\equiv M_1\left(\lambda\right), \end{aligned}$$

with $x\left(r,\lambda_0\right) = \left(r, \frac{1}{2}r^2\right)$, $H\left(\lambda\right) = diag(H_1\left(\lambda_0,\lambda_1\right), \ldots, H_j\left(\lambda_{k-1},\lambda_k\right), \ldots, H_{m+1}\left(\lambda_m,\lambda_{m+1}\right))$, $L\left(\lambda\right) = \left(L_1\left(\lambda_0,\lambda_1\right)', \ldots, L_j\left(\lambda_{k-1},\lambda_k\right)', \ldots, L_{m+1}\left(\lambda_m,\lambda_{m+1}\right)'\right)'$ and $\lambda = \left(\lambda_0,\ldots,\lambda_{m+1}\right)'$, where $H_j\left(\lambda_{k-1},\lambda_k\right)$ and $L_j\left(\lambda_{k-1},\lambda_k\right)$ are defined in Lemma 1. In general, for $T_{k-1} < t \leq T_k$, $1 < k < m$, the process $\hat{\omega}^{-1}\hat{S}_t^*$ converges to

$$\begin{aligned} \hat{\omega}^{-1}\hat{S}_t^* &\Rightarrow W\left(r\right) - \left(x\left(\lambda_1,\lambda_0\right), x\left(\lambda_2,\lambda_1\right), \ldots, x\left(r,\lambda_{k-1}\right), 0_{1\times(m-k+1)l}\right) \times \\ &\quad H\left(\lambda\right)^{-1} L\left(\lambda\right) \\ &\quad +x\left(\lambda_1,\lambda_0\right) H_1\left(\lambda_0,\lambda_1\right)^{-1} \times \\ &\quad \left[H_1\left(\lambda_0,\lambda_1\right)^{-1} + H_{m+1}\left(\lambda_m,\lambda_{m+1}\right)^{-1}\right]^{-1} \times \\ &\quad \left[H_1\left(\lambda_0,\lambda_1\right)^{-1} L_1\left(\lambda_0,\lambda_1\right) - H_{m+1}\left(\lambda_m,\lambda_{m+1}\right)^{-1} L_{m+1}\left(\lambda_m,\lambda_{m+1}\right)\right] \\ &\equiv M_k\left(\lambda\right), \end{aligned}$$

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while for $T_m < t \leq T_{m+1}$, we have

$$
\begin{aligned}
\hat{\omega}^{-1}\hat{S}_t^* \;\Rightarrow\;& W(r) - \left(x(\lambda_1,\lambda_0), \ldots, , x(\lambda_m,\lambda_{m-1}), x(r,\lambda_m)\right) H(\lambda)^{-1} L(\lambda) \\
& + \left[x(\lambda_1,\lambda_0) H_1(\lambda_0,\lambda_1)^{-1} - x(r,\lambda_m) H_{m+1}(\lambda_m,\lambda_{m+1})^{-1}\right] \times \\
& \left[H_1(\lambda_0,\lambda_1)^{-1} + H_{m+1}(\lambda_m,\lambda_{m+1})^{-1}\right]^{-1} \times \\
& \left[H_1(\lambda_0,\lambda_1)^{-1} L_1(\lambda_0,\lambda_1) - H_{m+1}(\lambda_m,\lambda_{m+1})^{-1} L_{m+1}(\lambda_m,\lambda_{m+1})\right] \\
\equiv\;& M_{m+1}(\lambda).
\end{aligned}
$$

Using all these elements and the CMT, we can establish that the limit distribution of the restricted KPSS statistic $\eta^*_{i,j}(\lambda_{i,j})$ for the (i,j)-th pair of individuals is given by

$$
\begin{aligned}
\eta^*_{i,j}(\lambda_{i,j}) &= \hat{\omega}_{i,j}^{-2} T^{-2} \sum_{t=1}^{T} \hat{S}_{i,j,t}^{*2} \\
&= \hat{\omega}_{i,j}^{-2} T^{-2} \left[\sum_{t=1}^{T_{b,1}^{i,j}} \left(\sum_{l=1}^{t} \hat{\varepsilon}^*_{i,j,l} \right)^2 + \cdots + \sum_{t=T_{b,k-1}^{i,j}+1}^{T_{b,k}^{i,j}} \left(\sum_{l=1}^{t} \hat{\varepsilon}^*_{i,j,l} \right)^2 + \cdots + \right. \\
&\quad \left. + \sum_{t=T_{b,m_{i,j}}^{i,j}+1}^{T} \left(\sum_{l=1}^{t} \hat{\varepsilon}^*_{i,j,l} \right)^2 \right] \\
&\Rightarrow \int_0^{\lambda_{i,j,1}} M_{i,j,1}(\lambda_{i,j})^2 \, dr + \cdots + \int_{\lambda_{i,j,m_{i,j}}}^{1} M_{i,j,m_{i,j}+1}(\lambda_{i,j})^2 \, dr,
\end{aligned}
$$

provided that $\hat{\omega}_{i,j}^2 \xrightarrow{p} \omega_{i,j}^2$, where $\xrightarrow{p}$ denotes convergence in probability. As mentioned above, the proof follows entirely with minor modifications for the case where the deterministic component is given by a constant term with level shifts. Theorem 1 has been proved.

Table 1: Empirical size of the restricted panel data statistic

Panel A: Cross-section independent time series

		$\rho_{i,j} \sim U(0.5, 0.6)$				$\rho_{i,j} = 0.8$			
		Homogeneous		Heterogeneous		Homogeneous		Heterogeneous	
		$N = 20$	$N = 40$	$N = 20$	$N = 40$	$N = 20$	$N = 40$	$N = 20$	$N = 40$
QPPP	$T = 100$	0.091	0.110	0.085	0.113	0.174	0.255	0.129	0.221
	$T = 200$	0.074	0.074	0.064	0.062	0.099	0.143	0.090	0.121
	$T = 300$	0.063	0.065	0.061	0.060	0.096	0.110	0.085	0.084
TQPPP	$T = 100$	0.108	0.150	0.160	0.240	0.039	0.057	0.070	0.109
	$T = 200$	0.041	0.040	0.038	0.046	0.018	0.012	0.015	0.015
	$T = 300$	0.039	0.028	0.040	0.031	0.019	0.014	0.019	0.013

Panel B: Cross-section dependent time series

		$\rho_{i,j} \sim U(0.5, 0.6)$				$\rho_{i,j} = 0.8$			
		Homogeneous		Heterogeneous		Homogeneous		Heterogeneous	
		$N = 20$	$N = 40$	$N = 20$	$N = 40$	$N = 20$	$N = 40$	$N = 20$	$N = 40$
QPPP	$T = 100$	0.305	0.337	0.300	0.333	0.340	0.378	0.337	0.374
	$T = 200$	0.251	0.274	0.241	0.264	0.291	0.316	0.282	0.314
	$T = 300$	0.271	0.292	0.267	0.296	0.282	0.318	0.288	0.309
TQPPP	$T = 100$	0.495	0.533	0.524	0.597	0.455	0.493	0.458	0.529
	$T = 200$	0.420	0.450	0.489	0.537	0.402	0.432	0.425	0.477
	$T = 300$	0.432	0.476	0.495	0.541	0.397	0.439	0.424	0.470

Notes: QPPP: Qualified Purchasing Power Parity; TQPPP: Trend Qualified Purchasing Power Parity.

Table 2: Empirical power of the restricted panel data statistic

Panel A: $\rho_{i,j} \sim U(0.5, 0.6)$, Cross-section independent time series

		$\sigma_w^2 = 0.001$				$\sigma_w^2 = 0.01$				$\sigma_w^2 = 0.1$			
		Homog.		Heterog.		Homog.		Heterog.		Homog.		Heterog.	
	$T \backslash N$	20	40	20	40	20	40	20	40	20	40	20	40
Q	100	0.218	0.333	0.193	0.297	0.949	0.998	0.905	0.996	1	1	1	1
	200	0.648	0.873	0.623	0.845	1	1	1	1	1	1	1	1
	300	0.945	0.995	0.938	0.997	1	1	1	1	1	1	1	1
T	100	0.121	0.185	0.172	0.277	0.318	0.493	0.365	0.581	0.960	0.998	0.963	0.997
	200	0.106	0.117	0.092	0.116	0.824	0.98	0.799	0.969	1	1	1	1
	300	0.181	0.241	0.167	0.224	0.997	1	0.996	1	1	1	1	1

Panel B: $\rho_{i,j} \sim U(0.5, 0.6)$, Cross-section dependent time series

		$\sigma_w^2 = 0.001$				$\sigma_w^2 = 0.01$				$\sigma_w^2 = 0.1$			
		Homog.		Heterog.		Homog.		Heterog.		Homog.		Heterog.	
	$T \backslash N$	20	40	20	40	20	40	20	40	20	40	20	40
Q	100	0.313	0.354	0.320	0.352	0.315	0.352	0.317	0.350	0.316	0.356	0.317	0.348
	200	0.247	0.273	0.246	0.270	0.250	0.273	0.245	0.268	0.246	0.277	0.249	0.296
	300	0.267	0.291	0.259	0.293	0.269	0.291	0.263	0.295	0.269	0.290	0.262	0.304
T	100	0.484	0.510	0.519	0.587	0.484	0.513	0.521	0.593	0.483	0.513	0.519	0.583
	200	0.417	0.452	0.486	0.540	0.417	0.452	0.486	0.540	0.414	0.452	0.488	0.541
	300	0.426	0.478	0.500	0.537	0.427	0.479	0.496	0.537	0.431	0.472	0.500	0.548

Notes: Q: Qualified Purchasing Power Parity; T: Trend Qualified Purchasing Power Parity. Continues.

Table 2 (cont): Empirical power of the restricted panel data statistic

Panel C: $\rho_{i,j} = 0.8\ \forall i, j$, Cross-section independent time series													
		$\sigma_w^2 = 0.001$				$\sigma_w^2 = 0.01$				$\sigma_w^2 = 0.1$			
		Homog.		Heterog.		Homog.		Heterog.		Homog.		Heterog.	
	$T \backslash N$	20	40	20	40	20	40	20	40	20	40	20	40
Q	100	0.203	0.283	0.161	0.237	0.477	0.710	0.348	0.576	0.960	0.999	0.907	0.995
	200	0.221	0.339	0.185	0.283	0.892	0.992	0.838	0.978	1	1	0.999	1
	300	0.348	0.532	0.304	0.458	0.992	1	0.989	1	1	1	1	1
T	100	0.040	0.062	0.065	0.114	0.046	0.076	0.082	0.142	0.175	0.037	0.233	0.442
	200	0.022	0.013	0.022	0.022	0.071	0.091	0.061	0.084	0.690	0.933	0.649	0.892
	300	0.028	0.022	0025	0.020	0.218	0.315	0.175	0.266	0.988	1	0.982	1
Panel D: $\rho_{i,j} = 0.8\ \forall i, j$, Cross-section dependent time series													
		$\sigma_w^2 = 0.001$				$\sigma_w^2 = 0.01$				$\sigma_w^2 = 0.1$			
		Homog.		Heterog.		Homog.		Heterog.		Homog.		Heterog.	
	$T \backslash N$	20	40	20	40	20	40	20	40	20	40	20	40
Q	100	0.356	0.393	0.348	0.400	0.356	0.394	0.349	0.400	0.355	0.394	0.351	0.402
	200	0.290	0.311	0.282	0.316	0.290	0.312	0.282	0.318	0.288	0.311	0.282	0.321
	300	0.282	0.326	0.285	0.315	0.281	0.323	0.284	0.313	0.284	0.321	0.284	0.321
T	100	0.452	0.496	0.456	0.528	0.452	0.497	0.456	0.525	0.452	0.496	0.459	0.523
	200	0.407	0.443	0.417	0.475	0.409	0.443	0.416	0.476	0.407	0.444	0.418	0.479
	300	0.389	0.433	0.428	0.478	0.390	0.433	0.426	0.475	0.391	0.432	0.428	0.475

Notes: Q: Qualified Purchasing Power Parity; T: Trend Qualified Purchasing Power Parity.

Table 3: Proportion of rejections for the pairwise analysis. Non-restricted and restricted QPPP and TQPPP hypotheses

		Proportion of rejections of the null hypothesis using critical values at the 5% level of significance	10% level of significance
Non-restricted	QPPP	0.132	0.191
	TQPPP	0.353	0.426
	Mixed	0.338	0.404
Restricted	QPPP	0.315	0.518
	TQPPP	0.088	0.212
	Mixed	0.238	0.478
Non-restricted (comparison)	QPPP	0.130	0.204
	TQPPP	0.398	0.469
	Mixed	0.403	0.493

Notes: QPPP: Qualified Purchasing Power Parity; TQPPP: Trend Qualified Purchasing Power Parity.

Table 4: Panel data statistics for the pairwise analysis. QPPP and TQPPP cases

Panel A: Non-restricted

		Independence		CS demeaned		Bootstrap distribution			
		Test	p-val	Test	p-val	90%	95%	97.5%	99%
QPPP	$Z(\lambda)$ Hom.	-0.126	0.550	-1.501	0.933	9.573	11.090	12.510	14.242
	$Z(\lambda)$ Het.	5.027	0.000	1.577	0.057	6.695	7.813	8.831	10.082
TQPPP	$Z(\lambda)$ Hom.	11.033	0.000	8.868	0.000	17.263	19.740	22.371	25.803
	$Z(\lambda)$ Het.	9.721	0.000	9.768	0.000	10.646	11.646	12.552	13.649
Mixed	$Z(\lambda)$ Hom.	9.080	0.000	9.981	0.000	12.149	14.164	16.291	19.442
	$Z(\lambda)$ Het.	9.608	0.000	10.273	0.000	8.924	9.862	10.799	12.004

Panel B: Parity restrictions

		Independence		CS demeaned		Bootstrap distribution			
		Test	p-val	Test	p-val	90%	95%	97.5%	99%
QPPP	$Z^*(\lambda)$ Hom.	29.792	0.000	29.187	0.000	8.620	11.984	15.544	20.004
	$Z^*(\lambda)$ Het.	27.367	0.000	28.233	0.000	8.010	10.575	13.143	16.861
TQPPP	$Z^*(\lambda)$ Hom.	16.922	0.000	22.753	0.000	15.026	16.623	18.155	19.883
	$Z^*(\lambda)$ Het.	19.762	0.000	23.075	0.000	14.636	15.770	16.871	18.276
Mixed	$Z^*(\lambda)$ Hom.	45.394	0.000	39.327	0.000	53.025	54.480	55.768	57.299
	$Z^*(\lambda)$ Het.	15.278	0.000	20.136	0.000	13.222	13.832	14.412	15.162

Notes: Hom.: Homogenous; Het.: Heterogenous. 'Independence' refers to cross section independence; 'CS demeaned' refers to cross section demeaning of Levin et al. (2002); 'Bootstrap distribution' is the Maddala and Wu (1999) parametric bootstrap to accommodate cross section dependence. QPPP: Qualified Purchasing Power Parity; TQPPP: Trend Qualified Purchasing Power Parity.

Table 5: Timing and number of breaks for a representative city pairs. Non-restricted QPPP and TQPPP hypotheses

	QPPP						TQPPP			
	$T^i_{b,1}$	$T^i_{b,2}$	$T^i_{b,3}$	$T^i_{b,4}$	$T^i_{b,5}$	$T^i_{b,6}$	$T^i_{b,1}$	$T^i_{b,2}$	$T^i_{b,3}$	$T^i_{b,4}$
ATL-BOS	1930	1944	1961	1990			1934	1951	1966	1982
ATL-CHI							1932			
ATL-CIN	1930	1949	1986				1930	1977		
ATL-CLE	1930	1979					1930	1945	1979	
ATL-DET	1936	1949	1984				1931	1944	1985	
ATL-HOU	1930	1973	1986				1943	1978	1991	
ATL-KAN	1931	1946	1961	1985			1931	1957	1985	
ATL-LA							1930			
ATL-MIN	1930	1977					1943			
ATL-NY	1927	1949	1961	1970	1980	1989	1934	1951	1980	
ATL-PHI	1930	1943					1931	1954	1981	
ATL-PIT	1930						1932	1986		
ATL-POR	1937	1992					1937	1977	1990	
ATL-SF	1930	1959	1977	1991			1932	1958	1992	
ATL-SEA	1930	1959	1992				1945	1971	1985	
ATL-SL	1930	1943	1986				1931	1986		

Notes: ATL (Atlanta), BOS (Boston), CHI (Chicago), CIN (Cincinnati), CLE (Cleveland), DET (Detroit), HOU (Houston), KAN (Kansas City), LA (Los Angeles), MIN (Minneapolis), NY (New York), PHI (Philadelphia), PIT (Pittsburgh), POR (Portland), SF (San Francisco), SEA (Seattle), and SL (St. Louis).

Table 6: Timing and number of breaks for a representative city pairs. Restricted QPPP and TQPPP hypotheses

	QPPP						TQPPP			
	$T^i_{b,1}$	$T^i_{b,2}$	$T^i_{b,3}$	$T^i_{b,4}$	$T^i_{b,5}$	$T^i_{b,6}$	$T^i_{b,1}$	$T^i_{b,2}$	$T^i_{b,3}$	$T^i_{b,4}$
ATL-BOS	1930	1945	1961	1980			1929	1951	1967	1989
ATL-CHI										
ATL-CIN	1929	1950	1996				1927	1986		
ATL-CLE	1937	1996					1926	1945	1979	
ATL-DET	1937	1947	1983				1926	1936	1984	
ATL-HOU	1930	1976	1986				1930	1976	1986	
ATL-KAN	1931	1946	1961	1985			1931	1957	1985	
ATL-LA										
ATL-MIN	1931	1996								
ATL-NY	1930	1949	1961	1970	1980	1989	1927	1951	1978	
ATL-PHI	1926	1981					1926	1950	1980	
ATL-PIT							1926	1985		
ATL-POR	1937	1996					1937	1978	1990	
ATL-SF	1932	1959	1978	1996			1931	1958	1996	
ATL-SEA	1937	1959	1996				1930	1971	1983	
ATL-SL	1927	1943	1986				1926	1986		

Notes: ATL (Atlanta), BOS (Boston), CHI (Chicago), CIN (Cincinnati), CLE (Cleveland), DET (Detroit), HOU (Houston), KAN (Kansas City), LA (Los Angeles), MIN (Minneapolis), NY (New York), PHI (Philadelphia), PIT (Pittsburgh), POR (Portland), SF (San Francisco), SEA (Seattle), and SL (St. Louis).

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